

Task Model 1	<b>Prompt Features:</b> The student is prompted to graph a simple				
Desmanas Turney	function and show key features.				
Graphing	Stimulus Guidelines:				
Graphing	• Graphs in answer choices must be within a -20 to 20				
DOK Level 2	coordinate grid, unless otherwise specified.				
	Functions must be chosen so that Key Features fit on the				
F-IF.C.7	grid.				
Graph functions expressed	• Key Features are values that can be interchangeable on a				
symbolically and show key	per item basis, e.g., "Which of these is the x-intercept of the function $2''$				
features of the graph, by	The function?				
using technology for more	number or a decimal to the tenths place: see Stimulus				
complicated cases.	quidelines within task models.				
a. Graph linear and	Linear functions will:				
quadratic functions and	• be in the form of $f(x) = mx + b$				
show intercepts,	$\circ  0 \leq m \leq 10, \ 0 \leq x \leq 10, \ \text{and} \ 0 \leq b \leq 10$				
maxima, and minima.	Key Features for linear include:				
b. Graph square root,	o slope				
niecewise-defined	• v-intercept				
functions, including	<ul> <li>The guadratic function may take the following forms:</li> </ul>				
step functions and	a) $f(x) = ax^2 + bx + c$				
absolute value	b) $f(x) = a(x-h)^2 + k$				
functions.	C) $f(x) = a(x - r_1)(x - r_2)$				
c. Graph polynomial	Key Features for quadratic include:				
runctions, identifying	<ul> <li>X- intercepts and/or y-intercept</li> <li>increasing interval and/or decreasing interval</li> </ul>				
factorizations are	<ul> <li>nositive interval and/or negative interval</li> </ul>				
available, and showing	<ul> <li>vertex</li> </ul>				
end behavior.	o symmetries				
e. Graph exponential and	<ul> <li>end behavior</li> </ul>				
logarithmic functions,	Square Roots functions will:				
showing intercepts and	• take the form $f(x) = a\sqrt{x-h+k}$				
trigonometric functions	• a is 1 or -1				
showing period.	h and k must be chosen so that there are x- and				
midline, and amplitude.	<i>v</i> -intercepts (e.g. not a function like $f(x) = \sqrt{x-3} + 1$ )				
	Cube Roots functions will:				
Evidence Required:	• have the form $f(x) = a\sqrt[3]{x-h} + k$				
1. Students graph	<ul> <li><i>a</i> is 1 or −1</li> </ul>				
symbolically and show	$\circ$ h and k are single digit integers				
key features of the	Precewise functions will:     A have pieces that are linear, quadratic, square ree				
graph.	<ul> <li>Absolute Value functions will:</li> </ul>				
	• have the form $f(x) = a x - h  + k$				
Tools: None	o <i>a</i> is rational				
Accessibility Note:	<ul> <li>h and k are single digit integers</li> </ul>				
Graphing items are not	<ul> <li>Key Features for square root, absolute value, and</li> </ul>				
currently able to be	piecewise include:				
Brailled. Minimize the	<ul> <li>x-intercepts and/or y-intercepts</li> <li>increasing interval and/or decreasing interval</li> </ul>				
number of items developed	<ul> <li>positive interval and/or negative interval</li> </ul>				
to this TM.	<ul> <li>points on the graph that correspond to relative</li> </ul>				
<ul> <li>piecewise-defined functions, including step functions and absolute value functions.</li> <li>C. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> <li><b>Evidence Required:</b></li> <li>1. Students graph functions expressed symbolically and show key features of the graph.</li> <li><b>Tools:</b> None</li> <li><b>Accessibility Note:</b> Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.</li> </ul>	• y-intercept • y-intercept • y-intercept • The quadratic function may take the following forms: a) $f(x) = ax^2 + bx + c$ b) $f(x) = a(x - h)^2 + k$ c) $f(x) = a(x - r_1)(x - r_2)$ • Key Features for quadratic include: • x- intercepts and/or y-intercept • increasing interval and/or negative interval • positive interval and/or negative interval • positive interval and/or negative interval • oretex • symmetries • end behavior • Square Roots functions will: • take the form $f(x) = a\sqrt{x - h} + k$ • a is 1 or -1 • h and k are single digit integers • h and k must be chosen so that there are x- and y-intercepts (e.g. not a function like $f(x) = \sqrt{x - 3} + 1$ ) • Cube Roots functions will: • have the form $f(x) = a\sqrt[3]{x - h} + k$ • a is 1 or -1 • h and k are single digit integers • Piecewise functions will: • have the form $f(x) = a\sqrt[3]{x - h} + k$ • a is 1 or -1 • h and k are single digit integers • Piecewise functions will: • have the form $f(x) = a x - h  + k$ • a is rational • h and k are single digit integers • Key Features for square root, absolute value, and piecewise include: • x- intercepts and/or y-intercepts • increasing interval and/or decreasing interval • positive interval and/or negative interval • positive interval and/or negative interval • points on the graph that correspond to relative				



Task Model 1	maximum and/or relative minimum values of the function
Pesnonse Tyne:	<ul> <li>symmetries</li> </ul>
Graphing	o ond hohavior
Graphing	
DOK Loval 2	<ul> <li>Polyliolillais will.</li> <li>be factorable even the rational numbers or unitton in</li> </ul>
DOK Level 2	• De lactorable over the rational numbers of written in
	Kov Fosturos, for polynomials includer
F-IF.C./	• Key realures for polynomials include:
Graph functions expressed	• x- and y-intercepts (some polynomials will have only
symbolically and show key	one x-intercept, e.g. $f(x) = x^{3}-27$
features of the graph, by	<ul> <li>points on the graph that correspond to relative</li> </ul>
hand in simple cases and	maximum and minimum values of the function
using technology for more	<ul> <li>end behavior</li> </ul>
complicated cases.	Logarithmic functions:
a. Graph linear and	• must be in the form $f(x) = a\log(x \pm h) \pm k$ or
quadratic functions and	$f(x) = a \cdot ln(x \pm h) \pm k$
show intercepts,	Exponential functions:
maxima, and minima.	• must be in the form $f(x) = b^{x-h} \pm k$
b. Graph square root,	• where $1 < b \le 100$ , h and k are single digit integers.
cube root, and	Key Features for exponential and logarithmic functions
piecewise-defined	include:
functions, including	$\circ$ x- and y-intercepts
step functions and	<ul> <li>end behavior</li> </ul>
absolute value	<ul> <li>vertical asymptotes</li> </ul>
functions	<ul> <li>Functions must be chosen so that requested Key Features</li> </ul>
c Granb polynomial	evist: for example, some exponential functions do not
functions identifying	cross one of the axes such as $f(x) = 3^{x-1} \pm 4$ and $f(x) =$
zeros when suitable	log(r-4) + 2
factorizations are	$\log(x - 4) + 2$ .
available and showing	• Item difficulty can be adjusted via these example methods,
available, and showing	but die not innited to these methods.
ellu Dellavior.	<ul> <li>Linear, quadratic, absolute value, square root, cube</li> <li>vest, relyanziele, riscovice, loss it has a vest relyance to the second secon</li></ul>
e. Graph exponential and	root, polynomiais, piecewise, logarithmic, exponential.
logarithmic functions,	
showing intercepts and	
end behavior, and	<b>Stimulus:</b> The student is presented with a function and a
trigonometric functions,	coordinate grid.
showing period,	2
midline, and amplitude.	<b>Example Stem 1:</b> Given a linear function with a slope of $\frac{2}{3}$
	and a <i>y</i> -intercept of 2:
Evidence Required:	• Using the Add Arrow tool, draw a line on the coordinate
1. Students graph	grid to graph the function.
functions expressed	• Place a point on the line representing the x-intercept of
symbolically and show	the function.
key features of the	
graph.	<b>Example Stem 2:</b> Given the function $y = \frac{2}{3}x + 2$
	<b>Example Stein 2.</b> Given the function $y = \frac{3}{3}x + 2$ ,
Tools: None	Using the Add Arrow tool, draw a line on the coordinate
	grid to graph the function.
Accessibility Note:	• Place a point on the line representing the <i>x</i> -intercept of
Graphing items are not	the function.
currently able to be	
Brailled. Minimize the	<b>Example Stem 3:</b> Given the function $y = \frac{1}{2} 2x - 1  + 2$ ,
number of items developed	• Use the Add Arrow tool to create a graph that
to this TM.	represents the function.
	1



#### Response Type: Graphing

# DOK Level 2

# F-IF.C.7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- d. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- e. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- f. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- f. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

# **Evidence Required:**

 Students graph functions expressed symbolically and show key features of the graph.

# Tools: None

# Accessibility Note:

Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.



**Interaction:** The student will graph lines using the Add Arrow tool and/or plot points using the Add Point tool.

**Rubric:** (2 points) The student graphs the correct line and plots the point at the correct location that represents a key feature [e.g., Example Stem 1, draws a correct line and plots the *x*-intercept located at (-3, 0)].

(1 point) The student graphs the correct line or plots the point at the correct location that represents a key feature [e.g., Example Stem 1, draws a correct line OR plots the *x*-intercept located at (-3, 0)].







Task Model 1	<b>Prompt Features:</b> The student is prompted to graph a					
lask filder I	complicated function, using the calculator tool, and show key					
Response Type:	reatures.					
Graphing						
	Stimulus Guidelines: (same as TM1a)					
DOK Level 2						
	TM1b					
	<b>Stimulus:</b> The student is presented with a function and a					
F-IF.C.7	Stilluus. The student is presented with a function and a					
Graph functions	coordinate grid.					
expressed symbolically						
and show key features of	<b>Example Stem 1:</b> Given the function $y = -x^2 + x + 6$ ,					
the graph, by hand in	• Place a point on the coordinate grid to show each x-					
simple cases and using	intercept of the function.					
technology for more	<ul> <li>Place a point on the coordinate grid to show the</li> </ul>					
complicated cases	maximum value of the function					
complicated cases.						
quadratic functions	<b>Example Stem 2:</b> Given the function $y = \sqrt{x+4} - 1$ ,					
and show intercepts,						
maxima, and minima.	<ul> <li>Place a point on the coordinate grid to show each x-</li> </ul>					
b. Graph square root,	intercept of the function					
cube root, and	• Place a point on the coordinate grid to show the v-					
niecewise-defined	intercent of the function					
functions including						
stop functions and						
step functions and						
absolute value	<b>Example Stem 3:</b> Given the function $y = \sqrt[3]{x-1+2}$ ,					
functions.	<ul> <li>Place a point on the coordinate grid to show the x-</li> </ul>					
c. Graph polynomial	intercept of the function.					
functions, identifying	• Place a point on the coordinate grid to show the y-					
zeros when suitable	intercept of the function.					
factorizations are						
available, and showing	[Retired example stems $4, 5, and 6$ ]					
end behavior						
e Granh exponential and	<b>Example Step 7.</b> Given the function $u = 0 \log(u + 4)$					
logarithmic functions	<b>Example Stem 7:</b> Given the function $y = 8\log(x + 4)$ ,					
	Place a point on the coordinate grid to show the x-					
	intercept of the function.					
and end benavior, and	<ul> <li>Place a point on the coordinate grid to show the y-</li> </ul>					
trigonometric	intercept of the function.					
functions, showing	у					
period, midline, and						
amplitude.	8-					
Evidence Required:	6+					
1. Students graph	4					
functions expressed						
symbolically and show						
key features of the						
graph						
grapin						
Tooler Calculator						
Manalan Old III						
version 3 Update:						
Retired example stems 4,						
5, and 6.						







Task Model 2	<b>Prompt Features:</b> Students are prompted to rewrite a guadratic to reveal the key features of its graph.
Response Type:	
Equation/Numeric	<b>Stimulus Guidelines</b> : The student is presented with a quadratic function used in a context. The quadratic function is:
DOK Level 2	<ul> <li>given in the form of ax<sup>2</sup> + bx + c,</li> <li>or in factored form if factorable, which is</li> </ul>
F-IF.C.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function: a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	$a(x - r_1)(x - r_2) \text{ or in completed square form, which is}$ $a(x - h)^2 + k, \text{ where } h = -b/2a \text{ and } k = c - b^2/4a$ <b>TM2a Stimulus:</b> The student is presented with a quadratic function. <b>Example Stem:</b> Enter an equation for the line of symmetry for the function defined by $f(x) = -8x^2 + 16x + 2$ . <b>Rubric:</b> (1 point) The student enters the correct equation (e.g., $x = 1$ ). <b>Response Type:</b> Equation/Numeric
Evidence Required: 2. Students write a quadratic function defined by an expression in equivalent factored form and completing the square form to reveal zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Tools: None	



#### Response Type: Matching Table

#### DOK Level 1

#### F-IF.C.8b

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function:

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions, such as y = $(1.02)^t$ ,  $y = (0.97)^t$ , y = $(1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.

#### **Evidence Required:**

3. Students write an exponential function defined by an expression in an equivalent form using the properties of exponents to reveal and explain different properties of the function and to classify them as representing exponential growth or decay.

Tools: None

**Prompt Features:** The student will use the properties of exponents to interpret expressions for exponential functions.

#### **Stimulus Guidelines:**

- Exponential functions will:
  - be given in the form of  $f(x) = ab^{(x-h)} + k$
  - *h* is a single digit integer
  - $\circ$  k is an integer, maximum value of 19
  - a = 1 or -1
  - *b* is a rational number, maximum value of 9; can be a non-repeating decimal.
- Key Features are values that can be interchangeable on a per item basis, e.g., "Which of these is the growth rate of the exponential function?"
- Key Features include:
  - o end behavior
  - rates of growth or decay
- The Key Feature being tested must correspond to a whole number or a decimal to the tenths place: see Stimulus guidelines within task models.
- Item difficulty can be adjusted to these example methods, but are not limited to these methods:
  - Interpreting growth vs. decay
  - Rewriting exponentials

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**Stimulus:** The student is presented with multiple functions.

**Example Stem:** Determine whether each function represents exponential growth or decay. Select the correct option for each function.

Function	Growth	Decay
$f(x) = \left(\frac{1}{2}\right)^x$		
$f(x) = \left(\frac{3}{2}\right)^{4x}$		
$f(x) = \left(\frac{7}{8}\right)^{4x}$		
$f(x) = \left(\frac{4}{3}\right)^{\frac{x}{12}}$		
$f(x) = 3\left(\frac{1}{3}\right)^{\frac{x}{12}}$		

#### **Rubric:**

(1 point) The student correctly sorts the exponential functions (e.g., Decay, Growth, Decay, Growth, Decay).



# Response Type: Matching Table

DOK Level 2

# F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## **Evidence Required:**

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

Tools: Calculator

**Prompt Features:** Students will identify the relationships, common properties, or key features shared between two functions represented in different ways.

# **Stimulus Guidelines:**

- Functions include: linear, quadratic, square root, cube root, piecewise-linear, absolute value, polynomial, exponential, and logarithmic functions.
- Key Features are values that can be interchangeable on a per item basis.
- Key Features include:
  - maximum and minimum values (for quadratic, piecewise-defined, absolute value, and polynomial functions)
  - end behavior (for square root and logarithmic functions: positive *x*-direction only)
  - x-intercepts and y-intercepts (for x-intercepts, not exponential; for y-intercepts, not logarithmic)
  - increasing and decreasing intervals
  - lines of symmetry
- The Key Feature being tested must correspond to a whole number or a decimal to the tenths place: see Stimulus guidelines within task models.
- Item difficulty can be adjusted to these example methods, but are not limited to these methods:
  - Functions come in table, graph or written description form.

# TM4a

**Stimulus:** The student is presented with two functions that must be represented in two different ways. Functions can be represented as a table of values, a graph, a function equation, or a written description.



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#### Task Model 4

#### Response Type: Matching Table

#### **DOK Level 2**

## F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

#### **Evidence Required:**

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

#### Tools: Calculator



2

f(x)



Select whether each statement is True or False about the given functions.

Statement	True	False
The minimum value of $f(x)$ is greater than the minimum value of $g(x)$ .		
The value of $x$ when $f(x)$ is at its minimum is less than the value of $x$ when $g(x)$ is at its minimum.		

#### **Rubric:**

(1 point) The student correctly identifies each statement as True or False (e.g., TT).

# HS Mathematics Item Specification C1 TM



Task	Model	4
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Response Type: Hot Spot

DOK Level 2

## F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## **Evidence Required:**

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

Tools: Calculator

# Accessibility Note:

Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM. **Prompt Features:** Students will select sections on a number line that represent an interval where two graphs have a shared key feature.

Stimulus Guidelines: (same as TM4a)

#### TM4b

**Stimulus:** The student is given two different functions (square root, cube root, piecewise-defined, or absolute value) and a number line representing the *x*-axis, and asked to indicate where the functions have a shared key feature.

**Example Stem:** In which interval(s) on the *x*-axis are the functions  $f(x) = \frac{1}{2}|2x| + 2$  and  $g(x) = -2x^2 + 12x - 16$  increasing? Click the interval(s) on the number line that represents where **both** functions are increasing.

<b>—</b> 1	1	T.	1	1		1	T.	T	1	1.0	-v
		- (	- (	-	- (	1	- (	- (			~~
-5-	-4-	-3-	-2-	-1	0	1	2	ک	4	5	

**Interaction:** The student will click on intervals on the number line using Hot Spots.

#### **Rubric:**

(1 point) The student clicks on the correct intervals (e.g., [1, 3]).



**Response Type:** Hot Spot

# HS Mathematics Item Specification C1 TM



Task Model 4

# Response Type: Matching Table

# DOK Level 2

# F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## **Evidence Required:**

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

#### Tools: None

**Prompt Features:** Students will identify the relationships, common properties, or key features shared between two functions represented in different ways.

# Stimulus Guidelines: (same as TM4a)

#### TM4c

**Stimulus:** The student is presented with the graph of a quadratic function and a table of equations that may or may not represent the function.

**Example Stem:** Determine whether each equation in the table represents the graph of the function shown. Select Yes or No for each equation.



Function	Yes	No
f(x) = (x - 3)(x - 9)		
f(x) = (x+3)(x-9)		
f(x) = (x+6)(x-9)		
$f(x) = (x - 3)^2 - 18$		
$f(x) = (x - 6)^2 - 9$		

# **Rubric:**

(1 point) Student correctly selects the functions that could be represented by the given graph (e.g., YNNNY).



# Response Type: Matching Table

## DOK Level 2

# F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## **Evidence Required:**

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).



#### TM4d

**Stimulus:** The student is presented with three functions in various forms (graphs, table of values, etc.) and a matching table that includes the equations of the three functions.

**Note:** If tables are given, the ordered pairs should show key features (zeros, etc.).

**Example Stem 1:** Select the appropriate box to indicate the match of each graph to its equation.





#### Response Type: Matching Table

DOK Level 2

# F-IF.C.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

#### **Evidence Required:**

Tools: None

4. Students compare properties of two functions represented in different ways (e.g., as equations, tables, graphs, or written descriptions).

Equation	Graph A	Graph B	Graph C
$f(x) = x\sqrt{3}$			
$f(x) = 3\sqrt{x}$			
$f(x) = \sqrt{3x}$			

#### **Rubric:**

(1 point) The student correctly matches the functions with the graph (e.g., Table A, Table C, Table B).

**Example Stem 2:** Select the appropriate box to indicate the match of each table of values to its equation.

Tab	le A	Table B		Tab	le C
x	<i>f</i> ( <i>x</i> )	x	<i>f</i> ( <i>x</i> )	x	f(x)
1	1.73	1	1.73	1	3.00
2	3.46	2	2.45	2	4.24
4	6.92	4	3.46	4	6.00
6	10.38	6	4.24	6	7.35
8	13.84	8	4.90	8	8.49

Equation	Table A	Table B	Table C
$f(x) = x\sqrt{3}$			
$f(x) = 3\sqrt{x}$			
$f(x) = \sqrt{3x}$			

#### **Rubric:**

(1 point) The student correctly matches the functions with the table (e.g., Table A, Table C, Table B).